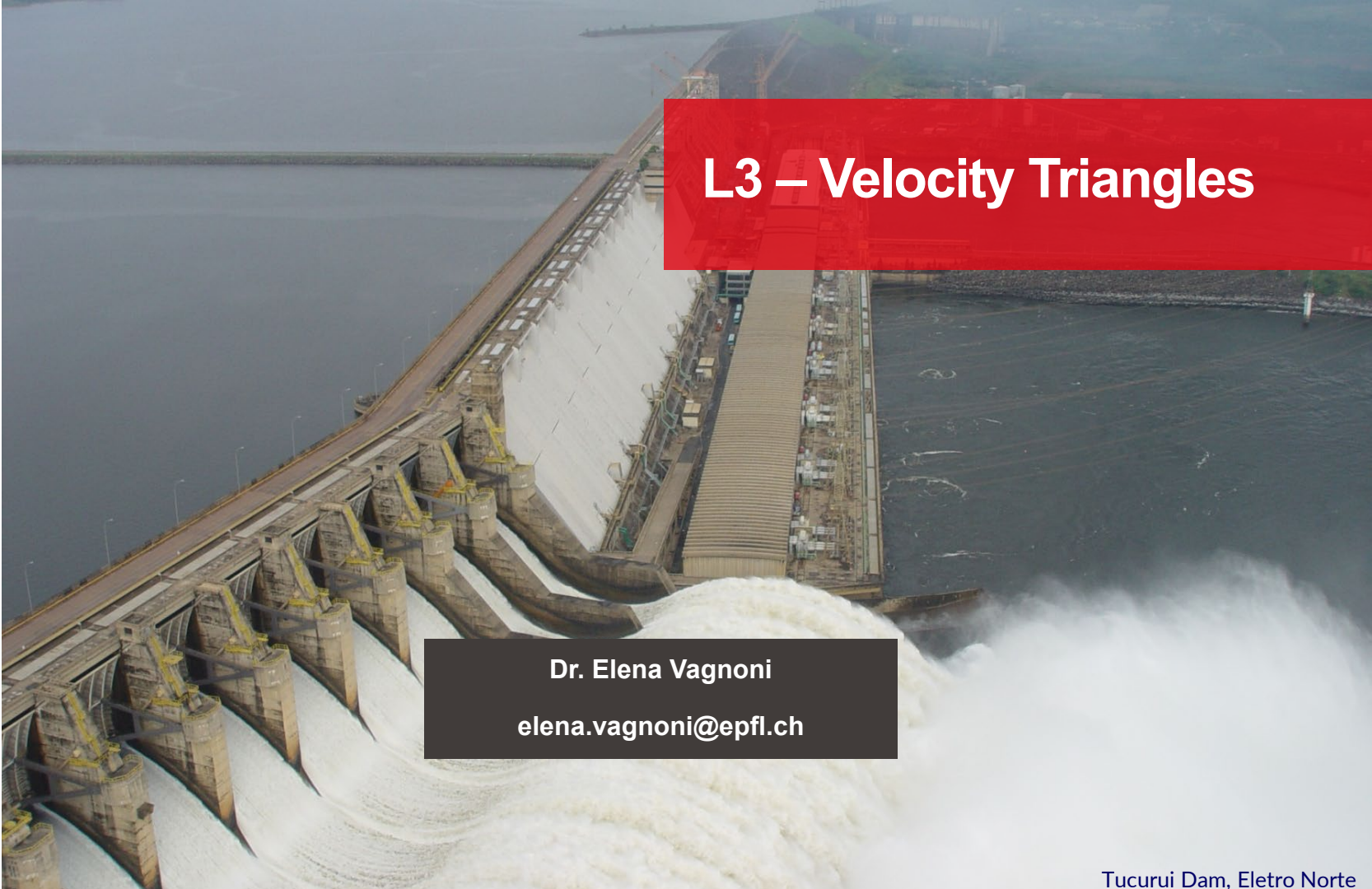


The background of the slide is an aerial photograph of the Tucuruí Dam. The dam is a long, concrete structure with multiple spillways. Water is cascading over the spillways, creating a large plume of white foam. The surrounding area is flat and appears to be a reservoir or a large body of water. A red semi-transparent banner is overlaid on the top right of the image.

L3 – Velocity Triangles

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EPFL Topics of the lecture

- Flow field
- Runner balance
- Euler equation
- Velocity Triangles
- Runner and impeller characteristic curves

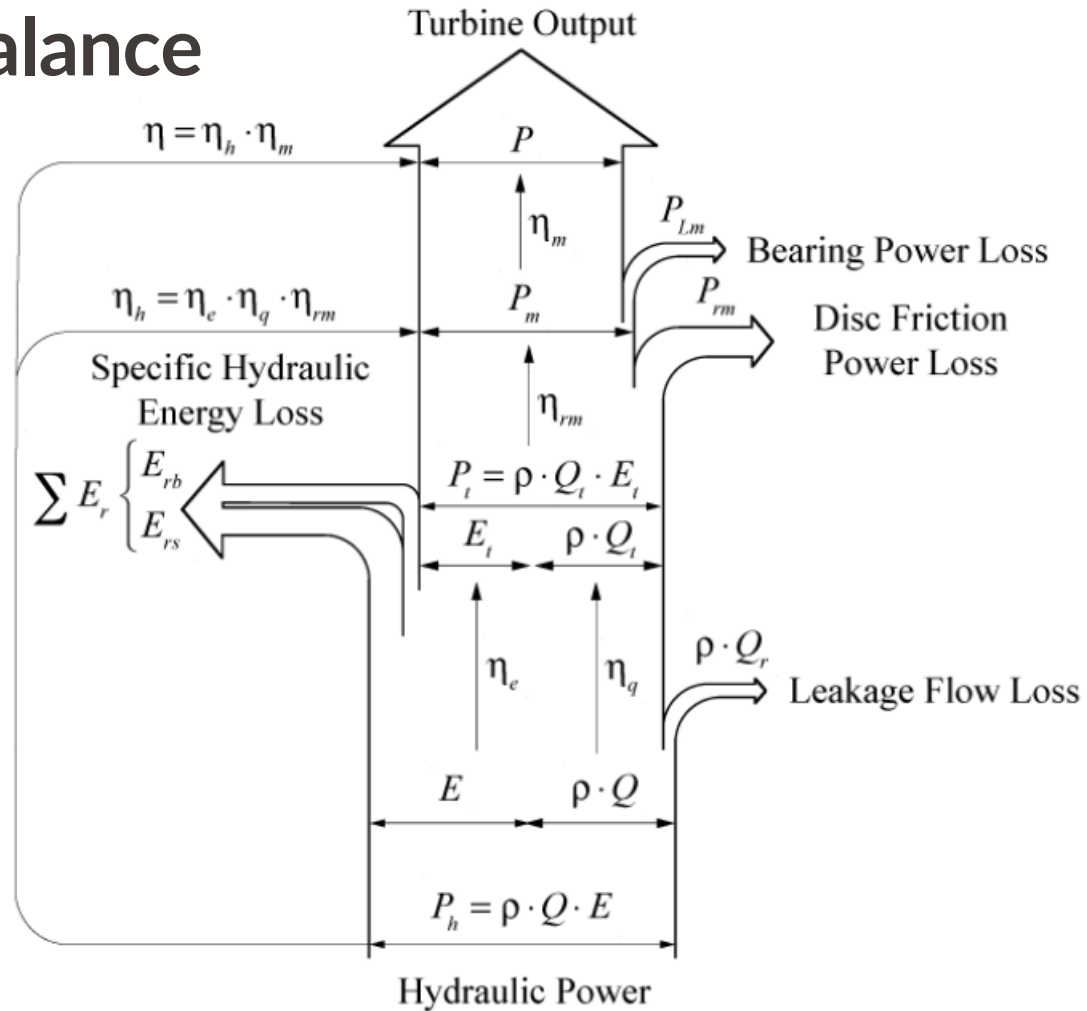
From L2: Power Balance

- Extracted Energy

$$E_t = \frac{P_t}{\rho Q_t}$$

- Driving Torque

$$P_t = \vec{\omega} \cdot \vec{T}_t$$



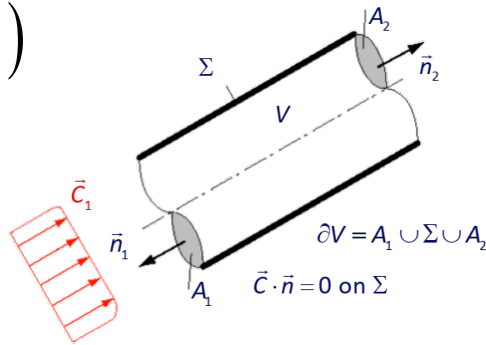
From L2: Specific Hydraulic Energy

- Local Mean Flow Specific Energy

$$h = \overbrace{\frac{p}{\rho}}^{\text{Pressure}} + \underbrace{g \times Z}_{\substack{\text{Gravity} \\ \text{Potential}}} + \overbrace{\frac{\vec{C}^2}{2}}^{\text{Kinetic}} + \text{Cste} \quad (\text{J} \cdot \text{kg}^{-1})$$

- Discharge

$$Q = \int_A \vec{C} \cdot \vec{n} dA \quad (\text{m}^3 \cdot \text{s}^{-1})$$



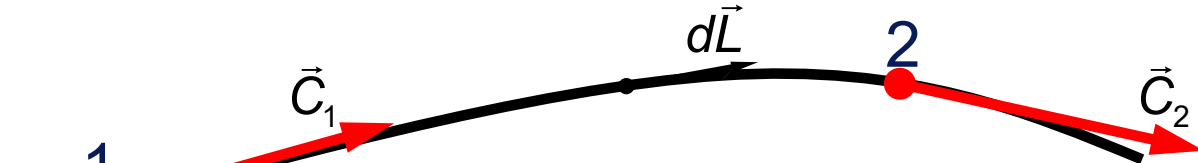
- Power

$$P_h = \int_A \rho h_t \vec{C} \cdot \vec{n} dA = \int_A \left(\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right) \rho \vec{C} \cdot \vec{n} dA \quad (\text{W})$$

From L2: Specific Hydraulic Energy

Specific Energy Balance

- Local Balance Along a Streamline



The diagram shows a curved black line representing a streamline. Two points, labeled 1 and 2, are marked with red dots. Red arrows labeled $\vec{C}_1$ and $\vec{C}_2$ indicate the flow direction at these points. A small black arrow labeled $d\vec{L}$ is shown along the streamline between the two points.

$$h_1 = h_2 - \int_{\vec{12}} \left[\vec{\nabla} \cdot \left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau_t}}}{\rho} \right) \right] \cdot d\vec{L} + \int_{\vec{12}} \frac{\partial \vec{C}}{\partial t} \cdot d\vec{L}$$

From L2: Specific Hydraulic Energy

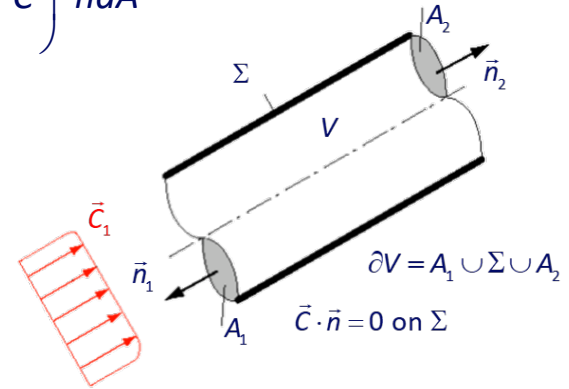
Mean Flow and Power Balance

- Flow Power Balance for a Pipe Domain V

$$\int_{A_1 \cup A_2} \rho h_t \vec{C} \cdot \vec{n} dA = - \int_V \overbrace{\frac{\partial}{\partial t} \left(\frac{\vec{C}^2}{2} \right)}^{\equiv 0 \text{ if Stationary}} \rho dV + \int_{A_1 \cup A_2} \overbrace{\left(\rho \left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho} \right) \cdot \vec{C} \right)}^{\equiv 0 \text{ if Homogeneous}} \cdot \vec{n} dA - \int_V (\Phi + \Pi) \rho dV \quad (\text{W})$$

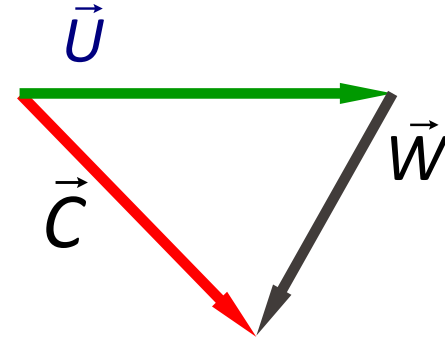
- Flow Power Dissipation

$$P_{r1 \div 2} = - \int_V \left(\underbrace{\overbrace{\frac{\Phi = \text{Viscosity}}{2\nu \overline{\overline{D}}}^{\equiv 2}} + \overbrace{\frac{\Pi = \text{Turbulence}}{\frac{\overline{\overline{\tau}}_t}{\rho} : \overline{\overline{D}}}^{\equiv}}}_{\text{Specific Power (W}\cdot\text{kg}^{-1})} \right) \rho dV$$



In an hydraulic turbine this is rotating $\rightarrow$ need to consider the relative flow!

EPFL Rotating Frame



- Absolute Flow Velocity

$$\vec{C} = \vec{U} + \vec{W}$$

- Rotating Velocity $\vec{U} = \vec{\omega} \times \vec{X}$

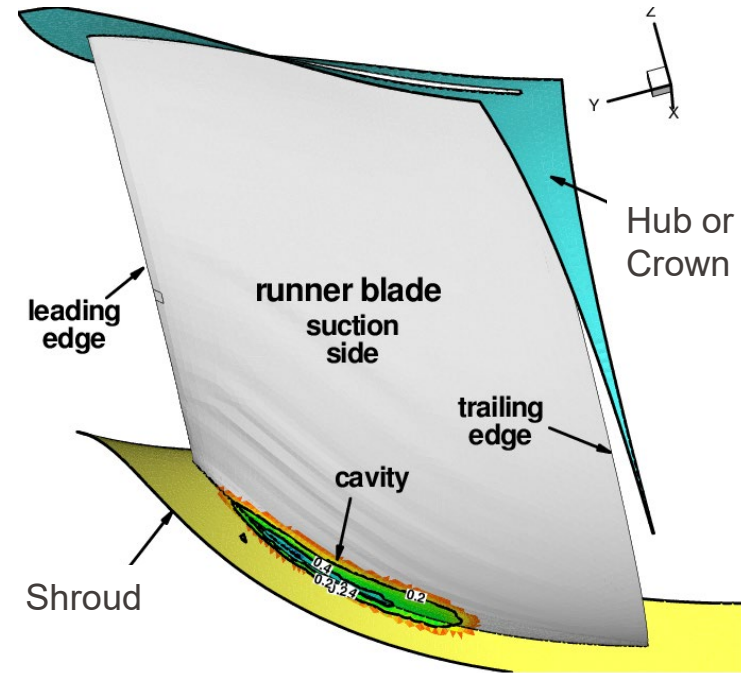
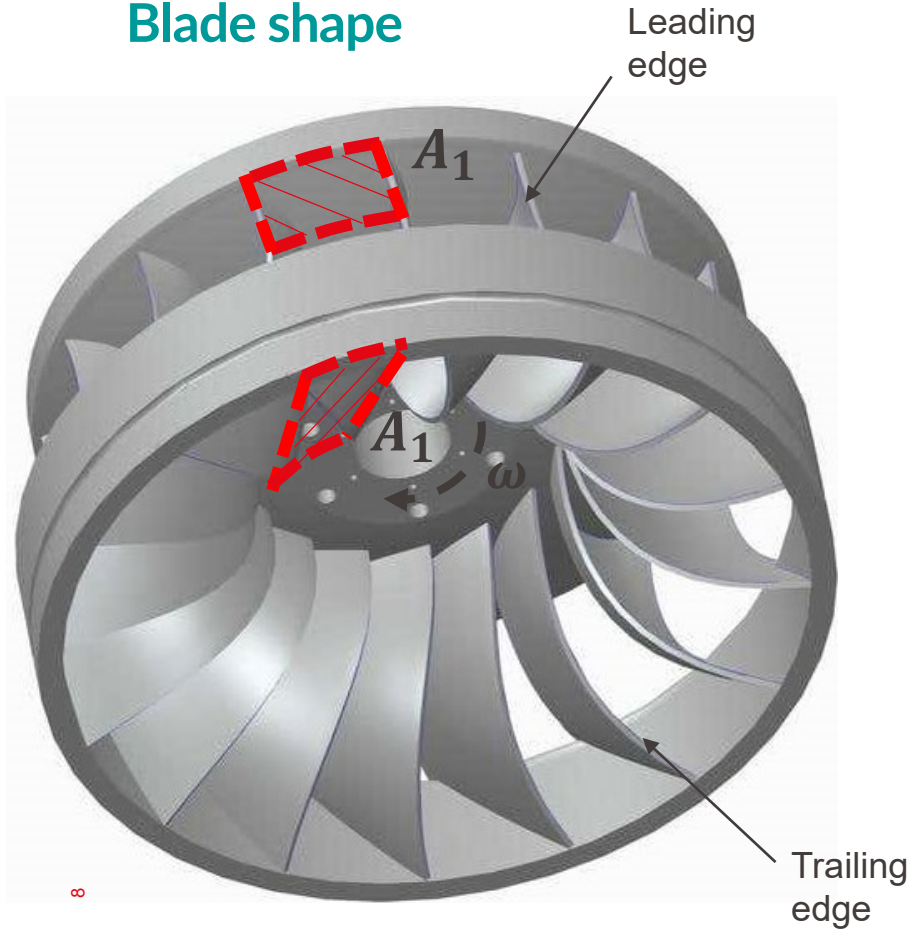
$$= \omega R$$

- Relative Flow Velocity

$$\vec{W} = \vec{C} - \vec{U}$$

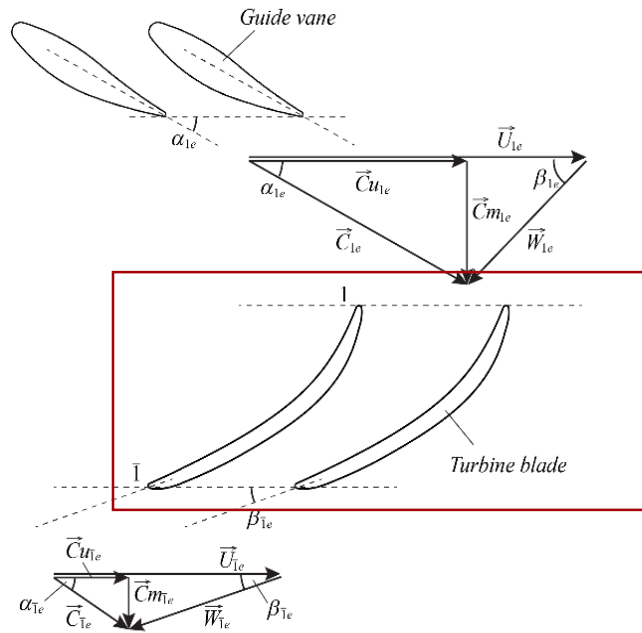
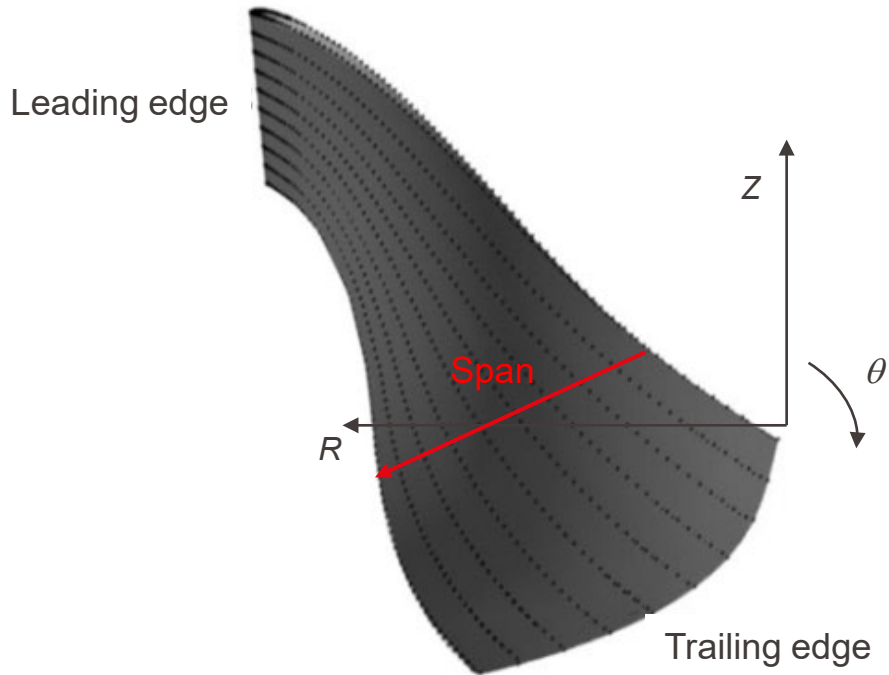
Rotating Frame

Blade shape



Blade to blade view

View of the blade channel projected on the 2D plane (θ, R) on a iso-span line

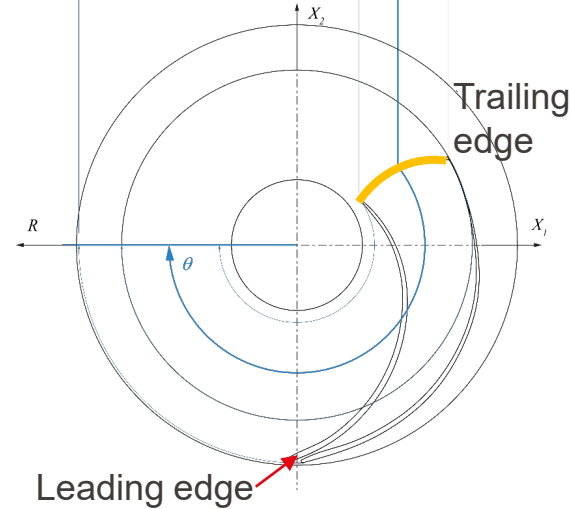
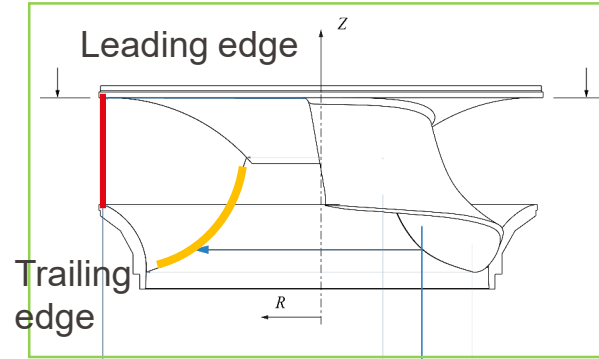
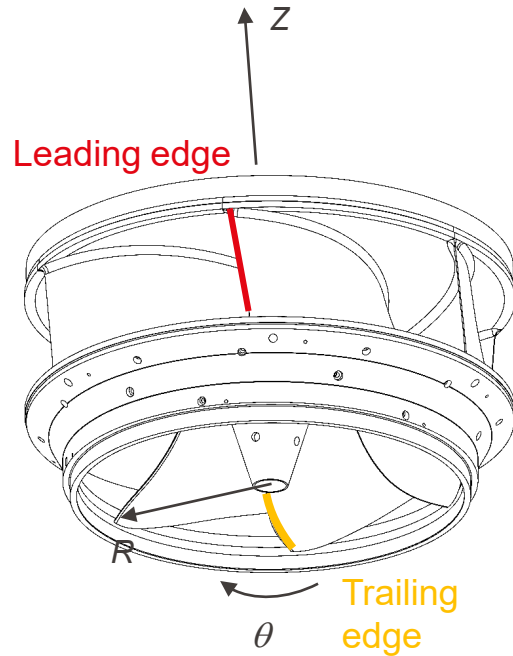


- Meridional Component $C_m = \frac{Q}{A}$
 - Tangential Component $C_u = \frac{C_m}{\tan \alpha} = U - \frac{C_m}{\tan \beta}$
- α Absolute flow angle: guide vanes
 β Relative flow angle: blade angle (hypotheses)

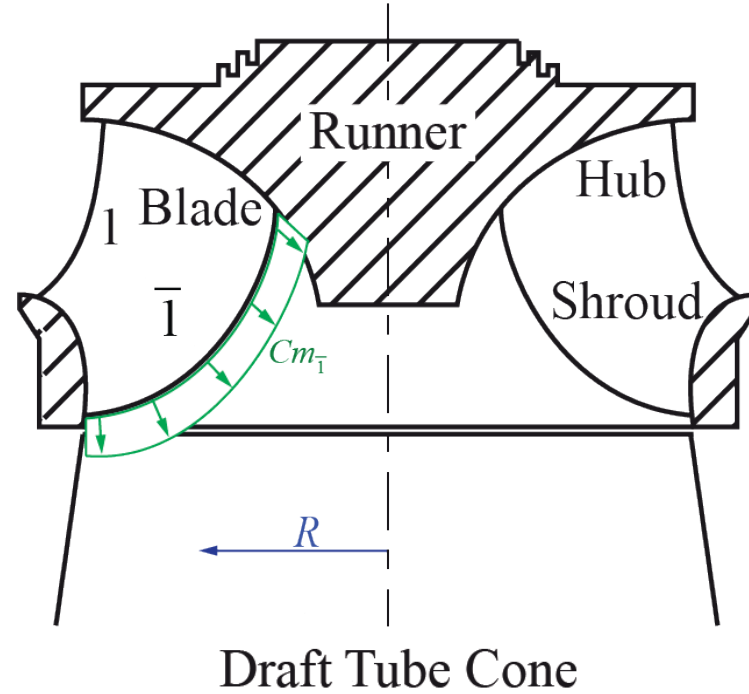
Meridional view

View of the blade projected on the 2D plane (Z, R)

Meridional view



Meridional View

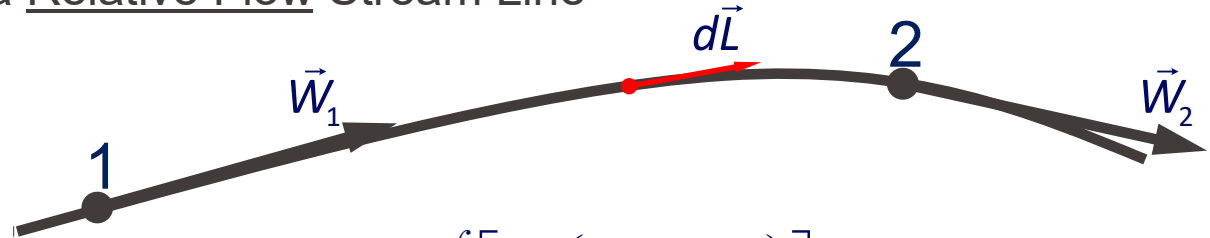


Specific Total Rothalpy

- Local rotational stagnation enthalpy: local rothalpy

$$h^{rel} = \frac{p}{\rho} + gZ - \frac{\vec{U}^2}{2} + \frac{\vec{W}^2}{2} + Cste \quad (\text{J} \cdot \text{kg}^{-1})$$

- Balance Along a Relative Flow Stream Line



$$h_1^{rel} = h_2^{rel} - \int_{\widehat{12}} \left[\vec{\nabla} \cdot \left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau_t}}}{\rho} \right) \right] \cdot d\vec{L} + \int_{\widehat{12}} \frac{\partial \vec{W}}{\partial t} \cdot d\vec{L}$$

Relative Flow Field

▪ Relative Acceleration:
$$\frac{D\vec{C}}{Dt} = \frac{D\vec{W}}{Dt} \Bigg|_{\text{Relative Frame}} - \underbrace{2\vec{\omega} \times \vec{W}}_{\text{Coriolis}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{X})}_{\text{Centrifugal}}$$

- Centrifugal Potential:

$$\begin{aligned} \vec{\omega} \times (\vec{\omega} \times \vec{X}) &= -\omega^2 \vec{R} = \vec{\nabla} \left(-\frac{\omega^2 R^2}{2} \right) \\ &= \vec{\nabla} \left(-\frac{\vec{U}^2}{2} \right) \end{aligned}$$

Relative Flow Field

- Continuity Equation $\vec{\nabla} \cdot \vec{W} = 0$
- Reynolds Averaged Navier-Stokes Equation

$$\begin{aligned} \frac{D\vec{C}}{Dt} &= \frac{D\vec{W}}{Dt} - \underbrace{2\vec{\omega} \times \vec{W}}_{\text{Coriolis}} + \underbrace{\vec{\nabla} \left(-\frac{\vec{U}^2}{2} \right)}_{\text{Centrifugal}} \\ &= -\vec{\nabla} \left(\underbrace{\frac{\overbrace{p}}{\underbrace{\rho}_{\text{Density}}}}_{\text{Pressure}} + \underbrace{gZ}_{\text{Potential}} \right) + \vec{\nabla} \cdot \left(\underbrace{2\nu \overline{\overline{D}}}_{\text{Visc. Stress}} + \underbrace{\frac{\overline{\overline{\tau_t}}}{\overline{\overline{\rho}}}}_{\text{Turb. Stress}} \right) \end{aligned}$$

$$\rightarrow \frac{D\vec{W}}{Dt} = \left[-\vec{\nabla} \left(\frac{p}{\rho} + gZ - \frac{\vec{U}^2}{2} \right) \right] + (2\vec{\omega} \times \vec{W}) + \vec{\nabla} \cdot \left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau_t}}}{\overline{\overline{\rho}}} \right)$$

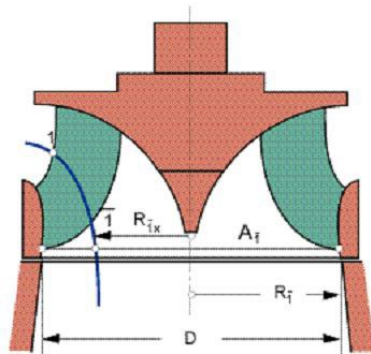
EPFL Turbine runner specific total rothalpy balance

- Rothalpy: $h^{rel} = \frac{p}{\rho} + gZ - \frac{\vec{U}^2}{2} + \frac{\vec{W}^2}{2} + Cste \quad (\text{J} \cdot \text{kg}^{-1})$

- Balance on a streamline: $h_1^{rel} = h_2^{rel} - \int_{12} \left[\vec{\nabla} \cdot \left(2\nu \vec{\overline{D}} + \frac{\vec{\overline{\tau}}_t}{\rho} \right) \right] \cdot d\vec{L} + \int_{12} \frac{\partial \vec{W}}{\partial t} \cdot d\vec{L}$

- Balance in the runner:
$$\int_{A_1 \cup A_{\bar{1}}} \left(\frac{p}{\rho} + gZ - \frac{\vec{U}^2}{2} + \frac{\vec{W}^2}{2} \right) \vec{W} \cdot \vec{n} dA = \int_{A_1 \cup A_{\bar{1}}} \left[\left(2\nu \vec{\overline{D}} + \frac{\vec{\overline{\tau}}_t}{\rho} \right) \cdot \vec{W} \right] \cdot \vec{n} dA +$$

$$- \int_V \left(\underbrace{\overline{\Phi}}_{\text{Viscosity}} = 2\nu \vec{\overline{D}} + \underbrace{\overline{\Pi}}_{\text{Turbulence}} = \frac{\vec{\overline{\tau}}_t}{\rho} : \vec{\overline{D}} \right) dV + \int_V \frac{\partial}{\partial t} \left(\frac{\vec{W}^2}{2} \right) dV$$

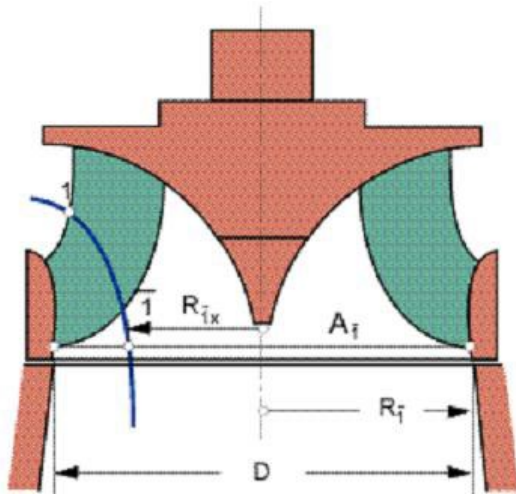


$$\partial V = A_1 \cup \Sigma_b \cup A_{\bar{1}}$$

EPFL Turbine runner hydraulic power balance

$$\int_{A_1} \left(\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right) \rho \vec{C}_1 \cdot \vec{n}_1 dA = \int_{A_{\bar{1}}} \left(\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right) \rho \vec{C}_{\bar{1}} \cdot \vec{n}_{\bar{1}} dA \boxed{+ P_t} \boxed{+ P_{rb}} +$$

$$- \int_{A_1 \cup A_{\bar{1}}} \rho \left[\left(2\nu \bar{\bar{D}} + \frac{\bar{\tau}_t}{\rho} \right) \cdot \vec{C} \right] \cdot \vec{n} dA + \boxed{\int_V \frac{\partial}{\partial t} \left(\frac{C^2}{2} \right) \rho dV}$$



$$\partial V = A_1 \cup \Sigma_b \cup A_{\bar{1}}$$

The change of hydraulic power of the mean flow through the runner-impeller is due to:

1. the power transfer between the flow and the runner ;
2. the rate of energy internal dissipation by both viscosity and turbulent production;
3. the net power budget of both viscous and turbulent stresses;
4. the rate of the mean flow kinetic energy change.

Turbine runner balance

- Rothalpy:
$$\int_{A_1 \cup A_{\bar{1}}} \left(\frac{p}{\rho} + gZ \right) \vec{C} \cdot \vec{n} dA = - \int_{A_1 \cup A_{\bar{1}}} \left(-\frac{\vec{U}^2}{2} + \frac{\vec{W}^2}{2} \right) \vec{C} \cdot \vec{n} dA + \int_{A_1 \cup A_{\bar{1}}} \left[\left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau}}}{\rho} \right) \cdot \vec{C} \right] \cdot \vec{n} dA - \int_V \left(\overbrace{2\nu \overline{\overline{D}}}^{\Phi=\text{Viscosity}} + \overbrace{\frac{\overline{\overline{\tau}}}{\rho} : \overline{\overline{D}}}^{\Pi=\text{Turbulence}} \right) dV + \int_V \frac{\partial}{\partial t} \left(\frac{\vec{W}^2}{2} \right) dV$$
- Power:
$$P_t + P_{rb} = - \int_{A_1 \cup A_{\bar{1}}} \left(\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right) \rho \vec{C} \cdot \vec{n} dA + \int_{A_1 \cup A_{\bar{1}}} \rho \left[\left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau}}}{\rho} \right) \cdot \vec{C} \right] \cdot \vec{n} dA - \int_V \frac{\partial}{\partial t} \left(\frac{\vec{C}^2}{2} \right) \rho dV$$

→

$$P_t = \int_{A_1 \cup A_{\bar{1}}} \left(-\frac{\vec{U}^2}{2} + \frac{\vec{W}^2}{2} - \frac{\vec{C}^2}{2} \right) \rho \vec{C} \cdot \vec{n} dA$$

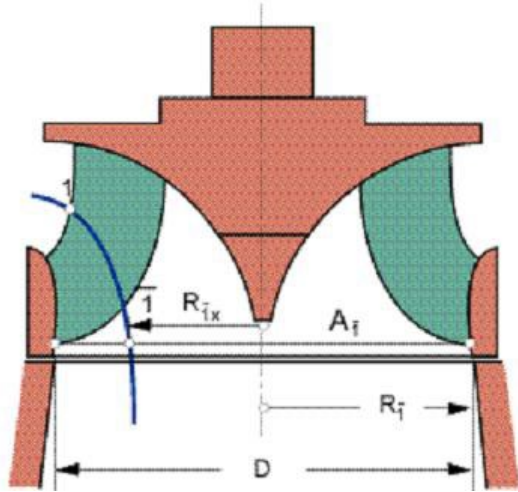
$$P_{rb} = \int_V \left(\overbrace{2\nu \overline{\overline{D}}}^{\Phi=\text{Viscosity}} + \overbrace{\frac{\overline{\overline{\tau}}}{\rho} : \overline{\overline{D}}}^{\Pi=\text{Turbulence}} \right) dV$$

Hypothesis:

- Fluxes of Reynolds and viscous stresses on the inlet and outlet section are neglected
- Steady flow

Energy Conversion in the Runner

Balance along a Relative Stream Line



- Energy Balance (e.g. Turbine)

$$gH_1 = gH_{\bar{1}} + E_t + E_{rb}$$

- Local Specific Energy Balance

$$E_t = [gH]_{\bar{1}}^1 - E_{rb}$$

$$= \left[\frac{p}{\rho} + gZ \right]_{\bar{1}}^1 + \left[\frac{\vec{C}^2}{2} \right]_{\bar{1}}^1 - E_{rb}$$

Energy Conversion in the Runner

Balance along a Relative Stream Line

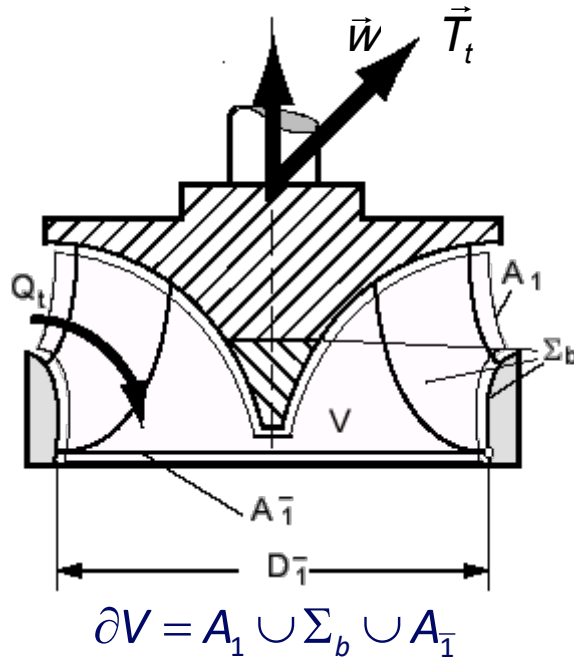
- Extracted (or transferred) Specific Energy

$$E_t = \left[\frac{\vec{C}^2}{2} + \frac{\vec{U}^2}{2} - \frac{\vec{W}^2}{2} \right]_{\bar{1}}^1 = \left[\frac{\vec{C}^2}{2} + \frac{\vec{U}^2}{2} - \frac{(\vec{C} - \vec{U})^2}{2} \right]_{\bar{1}}^1 = \vec{C}_1 \cdot \vec{U}_1 - \vec{C}_{\bar{1}} \cdot \vec{U}_{\bar{1}} = U_1 C u_1 - U_{\bar{1}} C u_{\bar{1}}$$

- Extracted power $P_t = \int_{A_1 \cup A_{\bar{1}}} \left(-\frac{\vec{U}^2}{2} + \frac{\vec{W}^2}{2} - \frac{\vec{C}^2}{2} \right) \rho \vec{C} \cdot \vec{n} dA = - \int_{A_1 \cup A_{\bar{1}}} (\vec{C} \cdot \vec{U}) \rho \vec{C} \cdot \vec{n} dA$

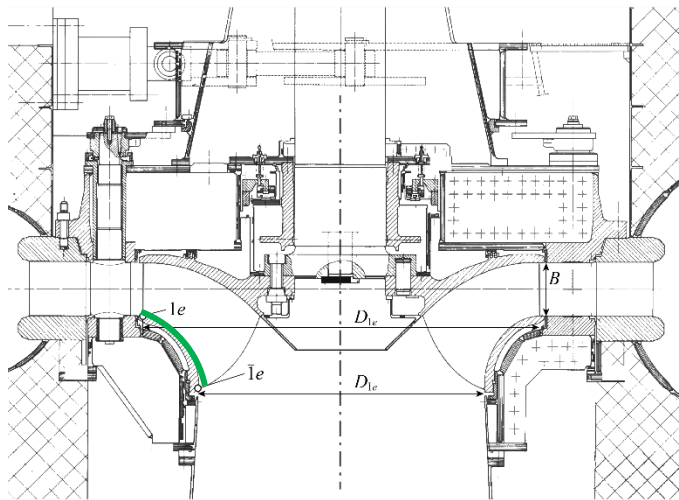
- Resulting torque $T_t = \frac{\rho Q_t E_t}{\omega} = \rho Q_t \frac{U_1 C u_1 - U_{\bar{1}} C u_{\bar{1}}}{\omega} = \rho Q_t (R_1 C u_1 - R_{\bar{1}} C u_{\bar{1}})$

Power Conversion in the Runner



■ Extracted Power

$$\begin{aligned}
 P_t &= \vec{\omega} \cdot \vec{T}_t \\
 &= - \underbrace{\int_{A_1 \cup A_{\bar{1}}} (\rho \cdot \vec{C} \cdot \vec{U}) \vec{C} \cdot \vec{n} dA}_{\text{Change of Angular Momentum Power (Mean Flow)}}
 \end{aligned}$$



- 1D Equation: local form of the transferred energy by considering any particular streamline, for example, the transferred specific energy is defined by using the outer, external, streamline between the 2 points:

$$E_t = k_{c_{u1e}} \vec{C}_{1e} \cdot \vec{U}_{1e} - k_{c_{u\bar{1}e}} \vec{C}_{\bar{1}e} \cdot \vec{U}_{\bar{1}e}$$

- As the Euler equation is defined for the mean flow, the local form uses the following flow velocity distribution coefficients to take into account the influence of the spatial velocity distribution :

$$k_{c_{ux}} = \left| \frac{\int (\vec{C} \cdot \vec{U}) \vec{C} \cdot \vec{n} dA}{Q_t (\vec{C}_x \cdot \vec{U}_x)} \right|$$

- Turbine Flow: Uniform Inlet & Solid Body Rotation Outlet.

Uniform flow at turbine inlet

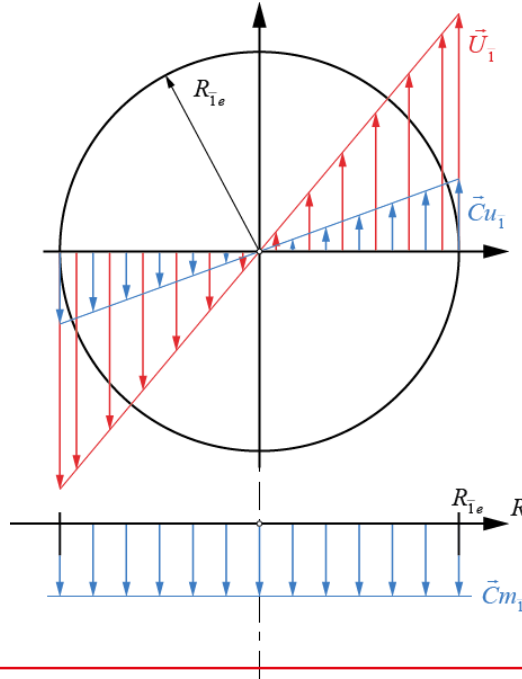
$$k_{C_{uA_{1e}}} = \left| \frac{\int_{A_{1e}} (\vec{C} \cdot \vec{U}) \vec{C} \cdot \vec{n} dA}{Q_t (\vec{C}_{1e} \cdot \vec{U}_{1e})} \right| = 1$$

$$k_{C_{mA_{1e}}} = \left| \frac{\int_{A_{1e}} \vec{C} \cdot \vec{n} dA}{A_{1e} C_{m_{1e}}} \right| = 1$$

Solid Body Rotation at turbine outlet

$$\frac{\vec{U}}{\vec{U}_{1e}} = \frac{R}{R_{1e}}$$

$$\frac{C_u}{C_{u1e}} = \frac{R}{R_{1e}}$$



$$k_{C_{uA_{1e}}} = \left| \frac{\int_{A_{1e}} (\vec{C} \cdot \vec{U}) \vec{C} \cdot \vec{n} dA}{Q_t (\vec{C}_{1e} \cdot \vec{U}_{1e})} \right|$$

$$k_{C_{mA_{1e}}} = \left| \frac{\int_{A_{1e}} \vec{C} \cdot \vec{n} dA}{A_{1e} C_{m_{1e}}} \right| = 1$$

$$E_t = (1) \times \vec{C}_{1e} \cdot \vec{U}_{1e} - \left(\frac{1}{2} \right) \times \vec{C}_{1e} \cdot \vec{U}_{1e}$$

→ Depending on the velocity components the extracted energy will be different!

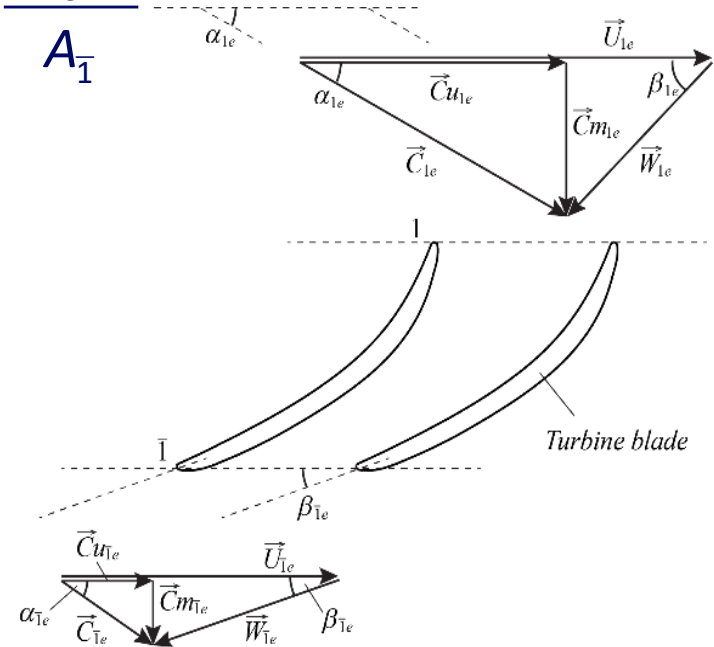
Turbine Runner Velocity Triangles and Characteristic Curve

$$E_t = k_{C_{u1e}} \vec{C}_{1e} \cdot \vec{U}_{1e} - k_{C_{u\bar{1}e}} \vec{C}_{\bar{1}e} \cdot \vec{U}_{\bar{1}e} = U_{1e} C_{u1e} - k_{C_{u\bar{1}e}} U_{\bar{1}e} C_{u\bar{1}e}$$

$$E_t = -k_{u\bar{1}e} U_{\bar{1}e}^2 + \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1} \frac{1}{\tan \alpha_{1e}} + k_{u\bar{1}e} \frac{1}{\tan \beta_{\bar{1}e}} \right] \frac{U_{\bar{1}e} Q_t}{A_{\bar{1}}}$$

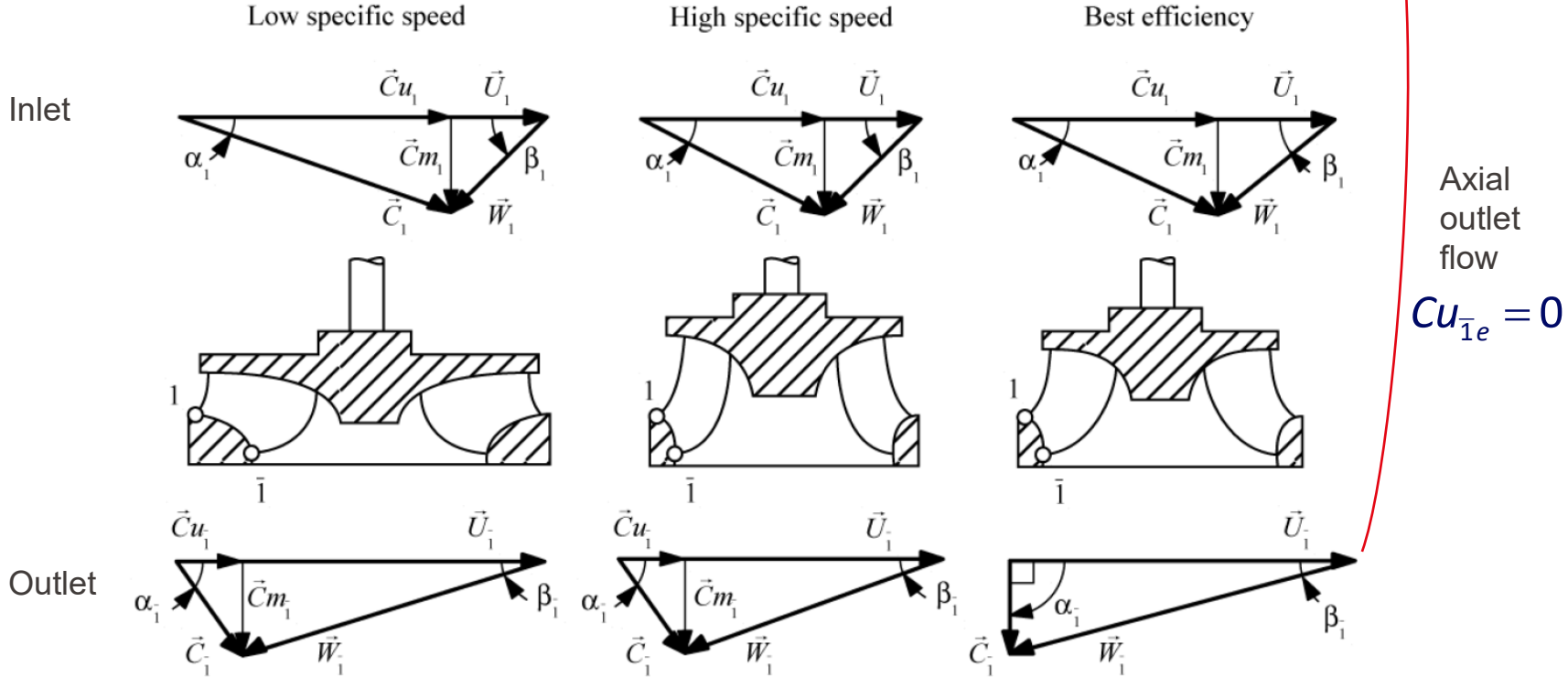
$$C_m = \frac{Q_t}{A}$$

$$C_u = \frac{C_m}{\tan \alpha} = U - \frac{C_m}{\tan \beta}$$

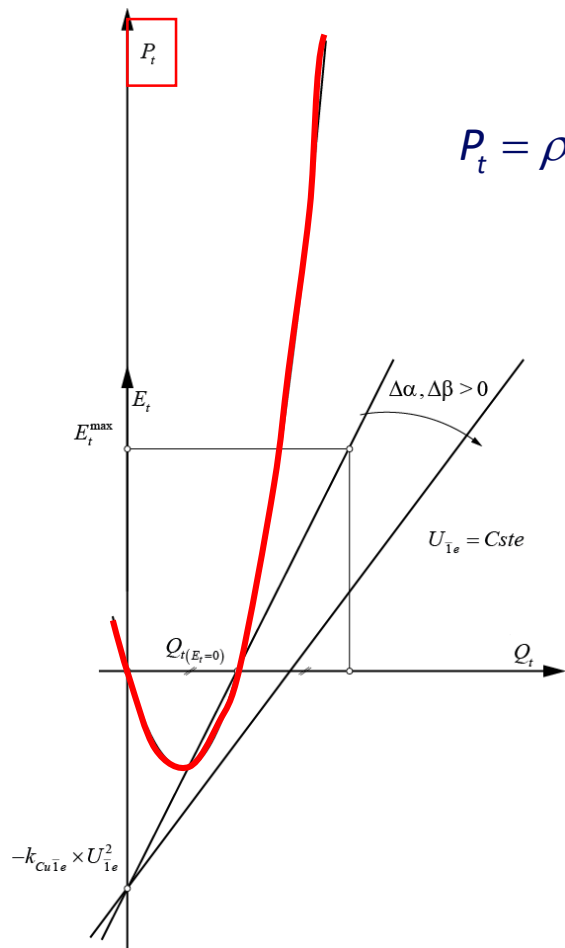


Turbine Runner Velocity Triangles

~~$$E_t = U_{1e} C u_{1e} - k_{C u_{1e}} U_{1e} C u_{1e}$$~~



Turbine Runner Characteristic Curve

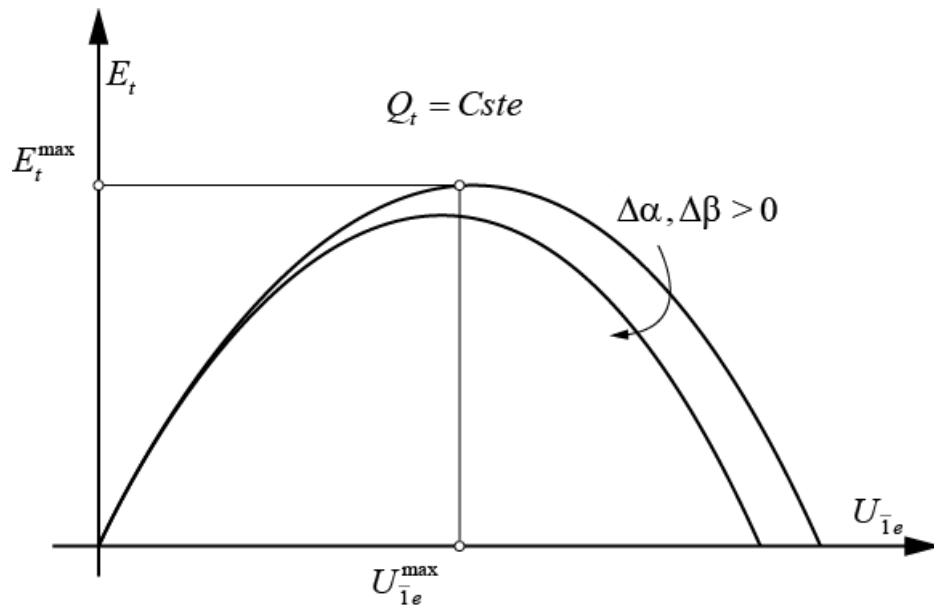


$$P_t = \rho Q_t E_t = -k_{u1e} U_{1e}^2 \rho Q_t + \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1 \operatorname{tg} \alpha_{1e}} + k_{u1e} \frac{1}{\operatorname{tg} \beta_{\bar{1}e}} \right] \frac{\rho U_{1e} Q_t^2}{A_{\bar{1}}}$$

$$E_t = -k_{u1e} U_{1e}^2 + \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1 \operatorname{tg} \alpha_{1e}} + k_{u1e} \frac{1}{\operatorname{tg} \beta_{\bar{1}e}} \right] \frac{U_{1e} Q_t}{A_{\bar{1}}}$$

Turbine Runner Characteristic Curve

$$E_t = -k_{u\bar{1}e} U_{\bar{1}e}^2 + \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1} \frac{1}{\operatorname{tg} \alpha_{1e}} + k_{u\bar{1}e} \frac{1}{\operatorname{tg} \beta_{\bar{1}e}} \right] \frac{U_{\bar{1}e} Q_t}{A_{\bar{1}}}$$



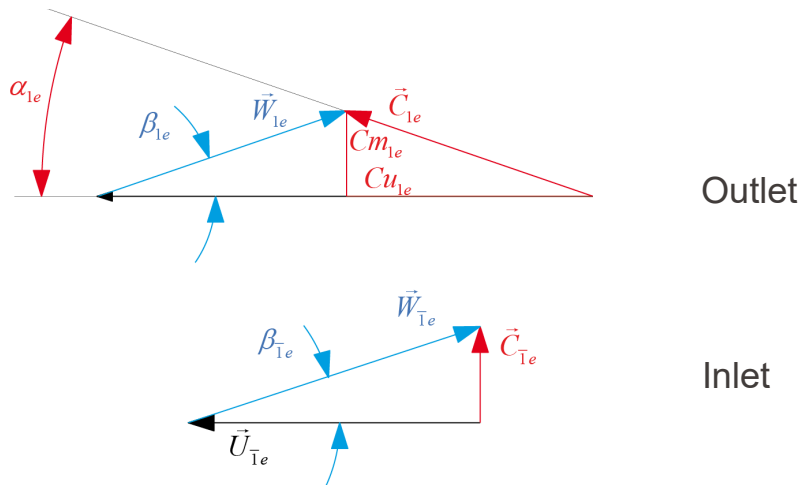
$$\frac{dE_t}{d\omega} = -2k_{u\bar{1}e} U_{\bar{1}e} + \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1} \frac{1}{\operatorname{tg} \alpha_{1e}} + k_{u\bar{1}e} \frac{1}{\operatorname{tg} \beta_{\bar{1}e}} \right] \frac{Q_t}{A_{\bar{1}}} = 0$$

$$U_{\bar{1}e}^{\max} = \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1} \frac{1}{\operatorname{tg} \alpha_{1e}} + k_{u\bar{1}e} \frac{1}{\operatorname{tg} \beta_{\bar{1}e}} \right] \frac{Q_t}{2k_{u\bar{1}e} A_{\bar{1}}}$$

$$E_t^{\max} = k_{u\bar{1}e} U_{\bar{1}e}^2$$

Pump Impeller Characteristic curve

Hypothesis: Swirl Free Inlet Flow $\rightarrow$ axial flow at the inlet section



$$E_t = k_{u1e} U_{1e}^2 - \left[k_{u1e} \frac{1}{\tan \beta_{1e}} + \underbrace{k_{u1e} \frac{R_{1e}}{R_{1e}} \frac{A_1}{A_{1e}} \frac{1}{\tan \alpha_{1e}}}_{=0 \text{ if } \alpha_{1e} = \frac{\pi}{2}} \right] U_{1e} \frac{Q_t}{A_1}$$

Pump Impeller Characteristic curve

Swirl Free Inlet Flow

- Transferred Power $P_t = \rho Q_t U_{1e} \left[k_{u1e} U_{1e} - k_{u1e} \frac{1}{\tan \beta_{1e}} \frac{Q_t}{A_1} \right]$

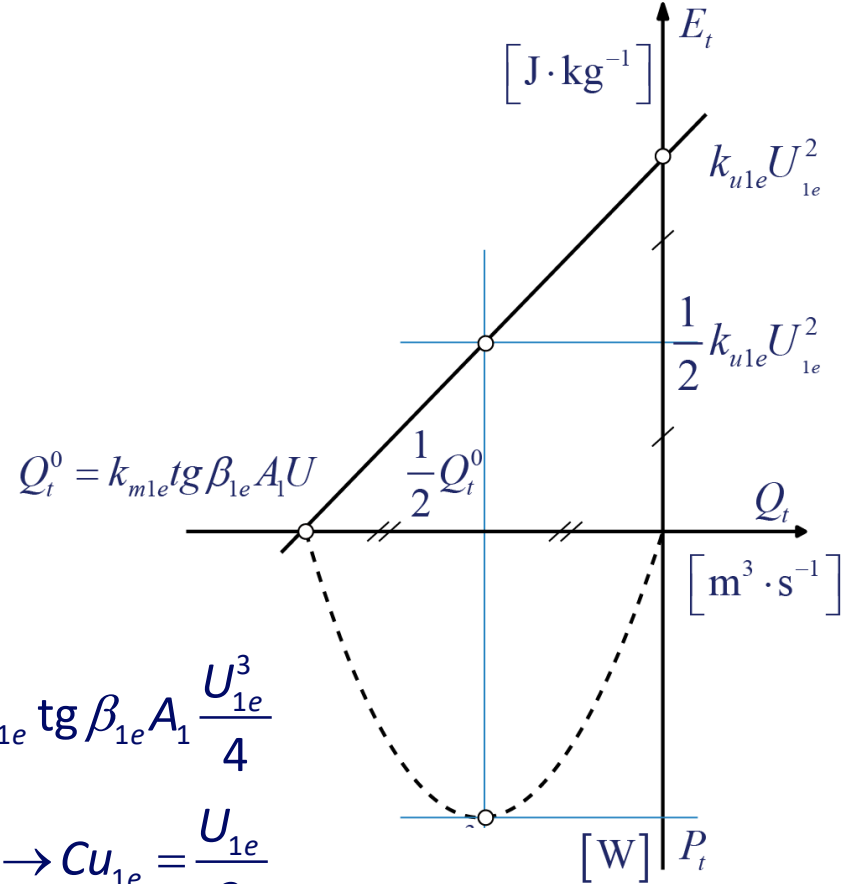
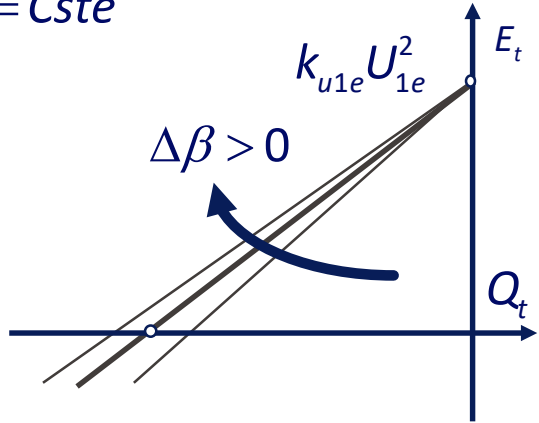
- No Load Conditions $P_t = 0$

$$Q_t = 0 \text{ and } Q_t^0 = \tan \beta_{1e} A_1 U_{1e}$$

Pump Impeller Characteristic curve

Swirl Free Inlet Flow

$\omega = Cste$

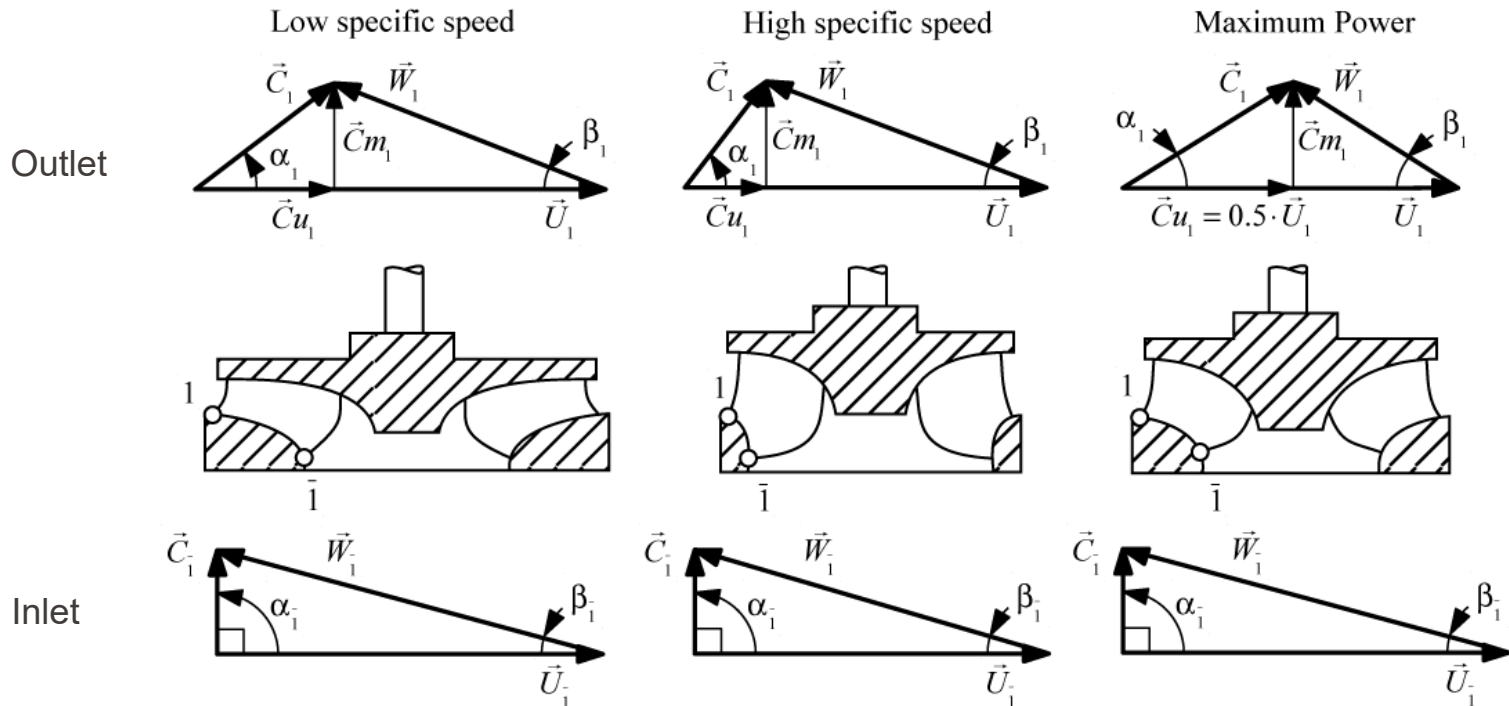


$$P_t = P_t^{\min} = \rho k_{u1e} \operatorname{tg} \beta_{1e} A_1 \frac{U_{1e}^3}{4}$$

$$E_t^{\min} = k_{u1e} \frac{U_{1e}^2}{2} \rightarrow Cu_{1e} = \frac{U_{1e}}{2}$$

Pump Impeller Velocity Triangles

Swirl Free Inlet Flow





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